SOLUTIONS

Joint Entrance Exam | IITJEE-2019

10th APRIL 2019 | Evening Session

Joint Entrance Exam | JEE Mains 2019

PART-A	PHYSICS

1.(4) Breaking stress $=\frac{F}{A}=\frac{F\times 4}{\pi d^2}$

$\Rightarrow \quad \frac{400\times 4}{\pi\times d^2}=379\times 10^6 \Rightarrow d=1.16\times 10^{-3} \quad =1.16\,\text{mm}$

2.(1) $m=Ia^2$ square loop

$m'=I\pi r^2$ circular loop

$4a=2\pi r\Rightarrow r=\frac{2a}{\pi}$ (radius of loop) $\quad\Rightarrow\quad m'=I\pi\left(\frac{2a}{\pi}\right)^2=\frac{4m}{\pi}$

3.() $U_i=V_f+\frac{1}{2}mv^2$

$\frac{9\times 10^9\times 10^{-12}}{10^{-3}}=\frac{9\times 10^9\times 10^{-12}}{9\times 10^{-3}}+\frac{1}{2}\times 4\times 10^{-9}\times v^2 \Rightarrow \quad v=6.32\times 10^4\, m/s$

No option match

4.(3) Beat frequency = $|f_1-f_2|=2\,Hz$

$\Delta t=\frac{1}{|f_1-f_2|}=0.5\,s$. Between two maximum.

5.(4) $Q=n\left(\frac{5R}{2}\right)\Delta T$

Q' (at constant pressure) = $nC_P\Delta T=n\times\frac{7R}{2}\Delta T \quad\Rightarrow\quad Q'=\frac{7Q}{5}$

6.(2) $\vec{L}=m(\vec{r}\times\vec{v})$

$\vec{r}=2t\hat{i}-3t^2\hat{j}\Rightarrow v=2\hat{i}-6t\hat{j} \quad =4\hat{i}-12\hat{j} \;=2\hat{i}-12\hat{j} \quad\Rightarrow\quad \vec{L}=-48\hat{k}$

7.(1) $a=\frac{-2.5\times 10^{-2}}{20\times 10^{-3}} \quad v^2=u^2+2as \;=1^2-\frac{2\times 2.5\times 10^{-2}}{20\times 10^{-3}}\times 20\times 10^{-2} \;=0.5$

$v\simeq 0.7\,\text{m/s}$

8.(4) $I_1:I_2=4:1$

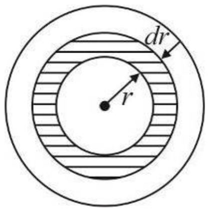
$\frac{I_{\max}}{I_{\min}}=\frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}=\left(\frac{3I}{I}\right)^2=\frac{9}{1}$

9.(3) $I_1=\frac{7M}{8}\times\frac{(2R)^2}{2}=\frac{7}{4}MR^2$

$I_2=\frac{2}{5}\times\left(\frac{M}{8}\right)\times\left(\frac{R}{2}\right)^2$ (Radius of new sphere = $\frac{R}{2}$) $=\frac{1}{80}MR^2$

$I_1:I_2=140$

10.(3) $R = \int dR = \int_a^b \frac{\rho dr}{4\pi r^2} = \frac{\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$



11.(2) $V_L = 6V \qquad i_L = \frac{6}{4\times10^3} = 1.5\times10^{-3}\,A$

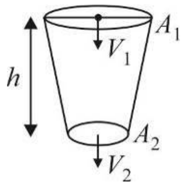
$V_B = 8V \qquad i_{Ri} = \frac{8-6}{1\times10^3} = 2\times10^{-3}$

$i_{zener} = 2\times10^{-3} - 1.5\times10^{-3} = 0.5\times10^{-3}\,A$

$V_B = 16V \qquad iR_i = \frac{16-6}{1\times10^3} = 10\times10^{-3}$

$i_{zener} = 10\times10^{-3} - 1.5\times10^{-3} = 8.5\times10^{-3}$

12.(4) $P_0 + \rho gh + \frac{1}{2}\rho v_1^2 = P_0 + \frac{1}{2}\rho v_2^2$



$\Rightarrow \qquad V_2^2 = V_1^2 + 2\text{gh}$

$V_2 = 2\,\text{m/s}$

$A_1V_1 = A_2V_2$

$\Rightarrow \qquad A_2 = \frac{10^{-4}}{2} = 5\times10^{-5}\,\text{m}^2$

13.(3) $N_A = N_0 e^{-5\lambda t} \qquad N_B = N_0 e^{-\lambda t}$

$\frac{N_A}{N_B} = \frac{1}{e^2} \Rightarrow \frac{e^{-5\lambda t}}{e^{-\lambda t}} = \frac{1}{e^2} \Rightarrow 4\lambda t = 2$

$t = \frac{1}{2\lambda}$

14.(3) $P_2 - P_1 = (d_2 - d_1)g\rho = 3.03\times10^6$

$d_2 - d_1 = \frac{3.03\times10^6}{10^3\times10} = 303\,\text{m} \sim 300\,\text{m}$

15.(4) e^- gets excited to $n = 4$

$3^2\left[\frac{13.6v}{1^2} - \frac{13.6}{4^2}\right] = \frac{nc}{\lambda} \qquad \Rightarrow \qquad \lambda = 10.5\,\text{nm}$

16.(4) $n = \frac{P}{\left(\frac{hc}{\lambda}\right)} = \frac{P\lambda}{hc} = \frac{2\times10^{-3}\times500\times10^{-9}}{6.6\times10^{-34}\times3\times10^8} \quad = 1.5\times10^{16}$

17.(1) $a^3\rho_b g = 0.3\times a^3\times\rho_\omega g \qquad \ldots\text{(i)}$

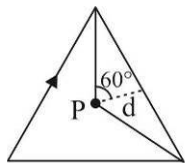
$a^3\rho_b g + mg = a^3\times\rho_\omega g \qquad \ldots\text{(ii)}$

$\Rightarrow \qquad m = \rho_\omega\times0.7\times a^3 = 10^3\times0.0875 \;= 87.5\,kg$

18.(3) $\alpha = \dfrac{\omega_f - \omega_i}{\Delta t} = \dfrac{25 \times 2\pi}{5} = 10\pi \dfrac{rad}{s^2}$

$\tau = I\alpha = \left(\dfrac{MR^2}{4} + MR^2\right)\alpha = \dfrac{5}{4}MR^2 = 1.96 \times 10^{-5} \simeq 2.0 \times 10^{-5}$ Nm

19.(4) $B_p = 3 \times \dfrac{\mu_0 i}{4\pi d}[\sin 60° + \sin 60°]$



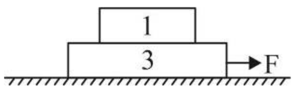
$d = \dfrac{1}{2\sqrt{3}} \qquad \Rightarrow \qquad B_p = 18 \times 10^{-6} T = 18 \mu T$

20.(2) $v_0 = \sqrt{\dfrac{GM}{r}} \Rightarrow \omega = \dfrac{v_0}{r} = \sqrt{\dfrac{GM}{r^3}} = \dfrac{2\pi n}{24 \times 60 \times 60} \quad \Rightarrow \qquad n = 11$

21.(3) $x = 5yz^2$

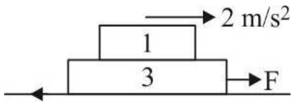
$y = \dfrac{x}{5z^2} \Rightarrow \quad [y] = \dfrac{[x]}{[z^2]} = \dfrac{M^{-1}L^{-2}T^4A^2}{M^2T^{-4}A^{-2}} = M^{-3}L^{-2}T^8A^4$

22.(3) max. acceleration of 1 kg block



$= \dfrac{f_{\max}}{1} = \dfrac{\mu \times 1 \times g}{1} = 2m/s^2$

When maximum F is applied on 3 kg.



$f_k = 0.2 \times 4 \times 10 = 8$

$F - 8 = (3+1) \times 2$

$F = 16N$

23.(2) Radiation pressure $P = \dfrac{I}{C}$

Momentum transferred

$= F \times \Delta t \quad = p \times A \times \Delta t$

$= \dfrac{I}{C} \times A \times \Delta t \quad = \dfrac{25 \times 10^4 \times 25 \times 10^{-4} \times 40 \times 60}{3 \times 10^{-8}}$

$= 5 \times 10^{-3} Ns$

24.(4) T = Time of flight $= \dfrac{24 \sin 15°}{g \cos 30°}$

Range Along the plane $R = 4\cos 15° \times T - \dfrac{1}{2} 9 \sin 30° \times T^2 \simeq 20\,\text{cm}$

25.(1) $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow m = \dfrac{v}{u} = 1 - \dfrac{v}{f}$

Slope $= \dfrac{-1}{f} = \dfrac{c}{b}$

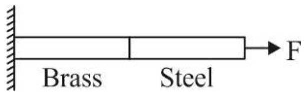
$|f| = \dfrac{b}{c}$

26.(1) $i = i_0(1 - e^{-Rt/L}) \Rightarrow 0.8 i_0 = i_0(1 - e^{-Rt/L})$

$$e^{-Rt/L} = 0.2 \Rightarrow \frac{Rt}{L} = \ell n5$$

$$t = \frac{L}{R}\ell n5 = \frac{10\times10^{-3}\times1.6}{(0.9+0.1)} = 0.016\text{ s}$$

27.() $$\Delta\ell = \frac{\frac{F}{Y_1A}}{\ell} + \frac{\frac{F}{Y_2A}}{\ell} = 0.2\times10^{-3}$$



$$\frac{F}{A}\left(\frac{\ell}{Y_1}+\frac{\ell}{Y_2}\right) = 0.2\times10^{-3}$$

$$\frac{F}{A}\left(\frac{3}{120\times10^{9}}\right) = 0.2\times10^{-3}$$

$$\frac{F}{A} = 8\times10^{-6}\frac{N}{m^2}$$

No option Match

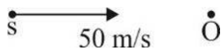
28.(4) $$V = V_0 \Rightarrow P_1 = \frac{P_0}{2}$$

$$V = 2V_0 \Rightarrow P_2 = \frac{7P_0}{8}$$

$$\Delta T = T_2 - T_1 = \frac{P_2V_2 - P_1V_1}{nR}$$

$$= \frac{\frac{7P_0}{8}\times2V_0 - \frac{P_0}{2}V_0}{nR} \quad = \frac{5P_0V_0}{4R}$$

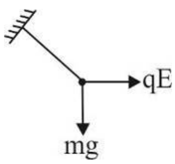
29.(3) $$f_{ap} = f\left(\frac{350}{350-50}\right) = 1000$$



f_{ap}' (after crossing)

$$= f\left(\frac{350}{350+50}\right) = \frac{1000\times3000}{350}\times\frac{350}{400} \quad = \frac{3000}{4} = 750\,\text{Hz}$$

30.(1) $$g_{eff} = \sqrt{g^2+\left(\frac{qE}{m}\right)^2}$$



$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$

PART-B	CHEMISTRY

1.(2) $pH = ?$ $\quad Kb = 10^{-9}$ $\quad \Rightarrow \quad P^{kb} = 5$ $\quad \Rightarrow \quad 10g2 = 0.301$

$0.02M\ NH_4Cl$

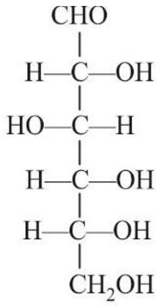
$\underset{\text{Acidic salt}}{NH_4Cl + H_2O} \qquad NH_4OH + HCl$

$pH = 7 - \frac{1}{2}\left(p^{kb} + \log C\right) \ = 7 - \frac{1}{2}\left(5 + \log 2 \times 10^{-2}\right) \ = 7 - \frac{1}{2}(3.301) \ = 7 - 1.6505$

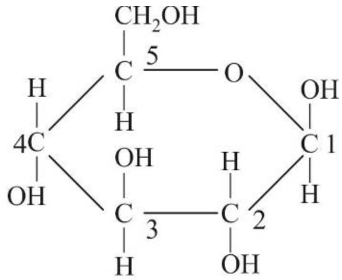
$pH = 5.3495$

2.(2) Glucose

Open chain



Closed



'4' $\Rightarrow$ no. of stereocentre '5' $\Rightarrow$ no. of stereocentre

3.(4) 'a' is correct

$MH_2 + 2H_2O \longrightarrow M(OH)_2 + 2H_2$

$MH + H_2O \longrightarrow MH + H_2$

(M is Alkali/Alkaline earth metal)

'b' is correct

$4BF_3 + 3LiAlH_4 \longrightarrow 2BrH_6 + 3LiF + 3AF_3$

'c' HF, CH_4 are called molecular hydrias

4.(4) $\underset{\text{(hydrated Solid)}}{X} \longrightarrow \underset{\text{(Monohydrate)}}{Y} \xrightarrow{T>373} \underset{\text{Anhydrous white Powder}}{Z}$

$\underset{\text{Washing Soda}}{Na_2CO_3 10H_2O} \longrightarrow Na_2CO_3.H_2O + 9H_2O \longrightarrow \underset{\text{Soda Ash}}{Na_2CO_3}$

5.(3) $K_1 = 2.5 \times 10^{-4}\ dm^3\ mol^{-1}\ s^{-1}\ T_1 = 327°C \Rightarrow 600K$

$K_2 = 1.0\ dm^3\ mol^{-1}\ s^{-1}\ T_2 = 527°C \Rightarrow 800K$

$E_A \Rightarrow$ in kJ / mole

Given $R = 8.314\ J / k\text{-mole}$

From $\log\frac{K_2}{K_1} = \frac{Ea}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\log\frac{1}{2.5} \times 10^{-4} = \frac{Ea}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\log\frac{1}{2.5 \times 10^{-4}} = \frac{Ea}{2.303 \times 8.314}\left(\frac{1}{600} - \frac{1}{800}\right)$

$E = 165.54\ kJ$

6.(2) CFSE

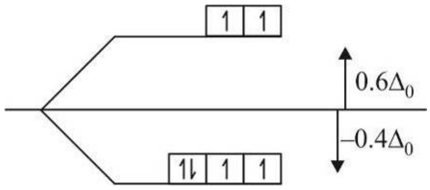
Of $\underset{\text{Octahedral Complex}}{\left[Fe(H_2O)_6\right]Cl_2}$

H_2O weak field Ligand

i.e. do not pair up the unpaired electron.

$\left[Fe(H_2O)_6\right]^{+2}$

$Fe^{+2}\ 1s^2\ 2s^2\ 2p^2\ 3s^2\ 3p^6\ 3d^6$.



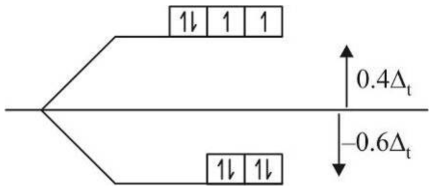
CFSE $\Rightarrow -4\times 0.4\Delta_0 + 2\times 0.6\Delta_0$

$= -1.6\Delta_0 + 1.2\Delta_0$

$= -0.4\Delta_0$

$K_2[NiCl_4]$

$\underset{Ni^{+2}}{[NiCl_4]^{-2}}$ $\underset{sp^3}{\text{Tetrahedral Complex}}$



$Cl^- \Rightarrow$ weak field Ligand do not pair up

Ni^{+2} $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^8$

$CFSE = -0.6\times 4\Delta_t + 0.4\times 4\Delta_t = -2.4\Delta_t + 1.6\Delta_t$

$CFSE = -0.8\Delta_t$

7.(1) Heptane

$C_7H_{16}(\ell) + 11\ O_{2(g)} \longrightarrow 7CO_{2(g)} + 8H_2O_{(\ell)}$

Since temperature T is given assume it at room temperature.

$\Delta H = \Delta U + \Delta n_{(g)}RT$

$\Delta H - \Delta U = -4RT$

$\Delta n_{(g)} = 7 - 11 = -4$

8.(1) Colloidal particles in lyophobic sols are charged particles which can move towards anode/cathode on application of electric field hence their precipitation can occure.

9.(1) Highest possible oxidation state of

Uranium $\rightarrow UO_2^{+2} \Rightarrow +6$

Plutonium $\rightarrow +7$ (Refer NCERT)

10.(2) Since in acylic compound angle strain is not present so option (2) is correct.

11.(2) Molar conductivity versus $\sqrt{c}$

K^+ is less hydrated than Na^+ so Λ_m for K^+ is more than Λ_m of Na^+.

12.(1) 12 five membered rings in C_{60} and 4 number trigons (triangles) in P_4 (white) phosphorous.

13.() R_n (Radon do not occur in nature)

It is obtained as a decay product of R_a

$\underset{\text{(Radium)}}{{}_{88}R_a^{226}} \longrightarrow \underset{\text{(Radon)}}{{}_{86}R_n^{222}} + {}_2He^4$

Technically no option is matching.

14.(3) Given $2SO_2(g)+O_2(g) \rightleftharpoons 2SO_3(g)$

$\Delta H=-57.2$ kJ/mol

$K_c=1.7\times10^{16}$

(1) From $K\downarrow = A\,e^{-\frac{\Delta H}{RT\uparrow}}$ as T increases, K decreases

(2) Equilibrium constant is not a affected by change in volume.

(3) Although K_c is large but it doesn't mean reaction go for completion.

(4) Equilibrium will shift in forward direction as pressure increases.

Hence (3) is incorrect.

15.(2) The correct order of first ionization enhalpies.

$$\Delta H_i \quad \underset{\underset{661}{\downarrow}}{Ti} < \underset{\underset{716}{\downarrow}}{Mn} < \underset{\underset{736}{\downarrow}}{Ni} < \underset{\underset{908\ (kJ/mol)}{\downarrow}}{Zn}$$

Hence option (2) is correct.

16.(2) $V_{mp}=\sqrt{\frac{2RT}{M}}$**,** where R $\Rightarrow$ universal gas constant $\Rightarrow$ Temperature $\Rightarrow$ Molar mass

Greater the $\frac{T}{M}$ ratio, greater will be the speed and higher the speed, the graph will shift towards right

17.(1) Aniline is a froth stabilizer.

18.(1) The gemstone, ruby has Cr^{3+} ions having repeating unit Al_2O_3Cr. The formula of beryl is $Be_3Al_2Si_6O_{18}$.

19.(3) Greater the stability of negative charge lesser will be the nucleophilicity. Neutral molecules are weaker nucleophiles as compared to the negatively charge species having same nucleophilic centre. Therefore order of nucleophilicity will be $H_2 < CH_3SO_3^- < CH_3CO_2^- < \bar{O}H$

20.(1) Refer NCERT

21.(1) $4Fe(s)+3O_2(g)\longrightarrow 2Fe_2O_3(g)$

No. of moles of O_2 required for one gram of $Fe=\frac{3}{224}$ moles

$2Mg(s)+O_2(g)\longrightarrow 2MgO(s)$

No. of moles of O_2 required for one gram of Mg $=\frac{1}{48}$ moles

$P_{4(s)}+5O_2\longrightarrow P_4O_{10(s)}$

No. of moles of O_2 required for one gram of $P_4=\frac{5}{124}$ moles

$C_3H_8(g)+5O_2(g)\longrightarrow 3CO_2(g)+4H_2O(\ell)$

No. of moles of O_2 required for one gram of $C_3H_8=\frac{5}{44}$ moles

So least amount is needs for the first reaction and is equal to $\frac{3}{224}\times32=0.4285$g

22.(4) The shortest wavelength for Lyman series

$$\frac{1}{\lambda_{\text{Lyman}}} = R_H\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$$

$$\lambda_{\text{Lyman}} = \frac{1}{R_H}$$

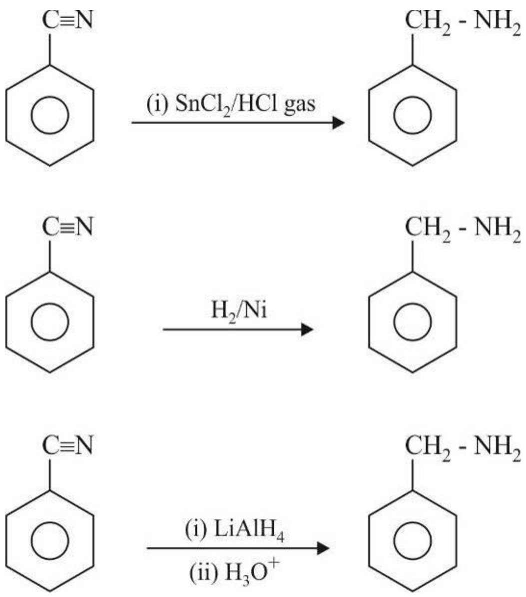
The shortest wavelength for paschen series

$$\frac{1}{\lambda_{\text{Paschen}}} = R_H\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right]$$

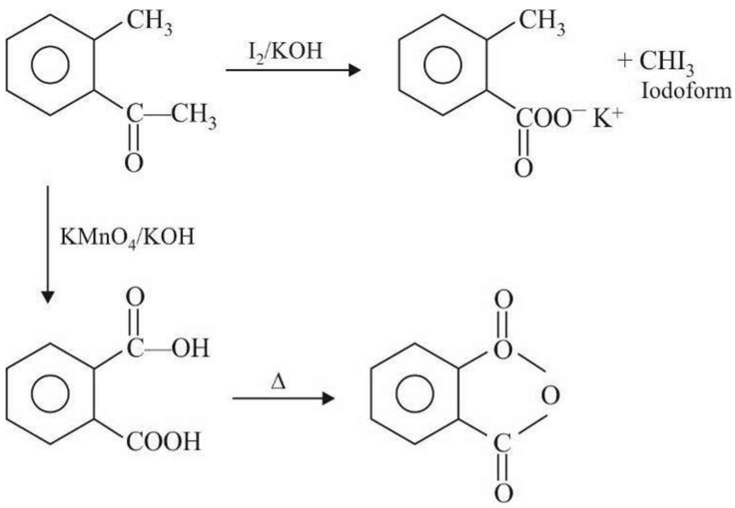
$$\lambda_{\text{Paschen}} = \frac{9}{R_H}$$

$$\frac{\lambda_{\text{Paschen}}}{\lambda_{\text{Lyman}}} = \frac{9/R_H}{1/R_H} = 9$$

23.(1)

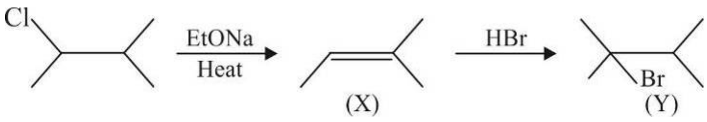


24.(4)



Phtholic Anhydride
used for preparation
of phenolphthalein

25.(3)



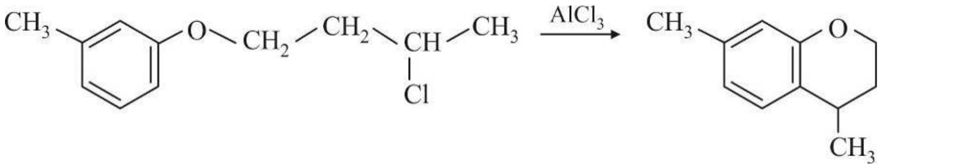
26.(2) Higher R_f value means higher volatile means low adsorption.

27.(2) $\Delta T_b = K_b \times m$

Since the molality for both 'A' and 'B' is same

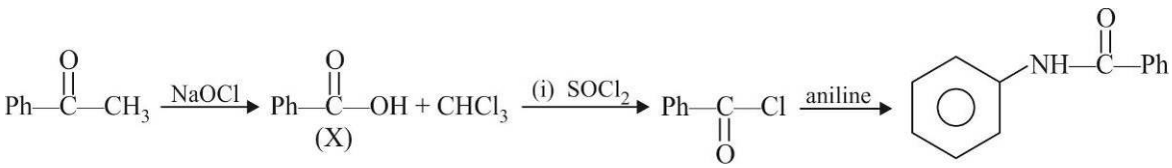
$\frac{\Delta T_b(A)}{\Delta T_b(B)} = \frac{K_b(A)}{K_b(B)} = \frac{1K}{5K} \Rightarrow 1:5$

28.(3) It is intermolecular friedel craft reaction and will go via formation of carbocation



29.(2) Air pollution that occurs in sunlight is oxidizing smog

30.(2)



PART-C	MATHEMATICS

1.(1) For given numbers mean $(\mu) = 16$

Standard deviation $(\sigma) = 4$

Mean of $(x_i - 4) = 16 - 4 = 12$

Standard deviation of $(x_i - 4) = 16$ (as standard deviation is not affected by shifting)

$\frac{\sum (x_i - 4)^2}{50} - (12)^2 = (16)^2$

$\frac{\sum (x_i - 4)^2}{50} = 256 + 144 = 400.$

2.(3) $5 + |2^x - 1| = 2^x(2^x - 2)$ solution will exist only for $x > 0$ as for $x < 0$ R.H.S. is negative.

For $x > 0$

Let $2^x = t$

$t^2 - 3t - 4 = 0$

$(t-4)(t+1) = 0$

$t = 4, t \neq -1$

$2^x = 4 \Rightarrow x = 2$ only one solution.

3.(4) $2x - y + 2z + 3 = 0$ …(1)

$4x - 2y + 4z + \lambda = 0 \Rightarrow 2x - y + 2z + \frac{\lambda}{2} = 0$ …(2)

Distance between (1) and (2)

$$\frac{\left|3-\frac{\lambda}{2}\right|}{3}=\frac{1}{3} \Rightarrow \left|3-\frac{\lambda}{2}\right|=1$$

$\lambda = 8$ or $\lambda = 4$

$2x - y + 2z + \mu = 0$ ...(3)

Distance between (1) and (3)

$$\frac{|3-\mu|}{3}=\frac{2}{3} \quad \Rightarrow \quad 3-\mu=2 \text{ or } 3-\mu=-2$$

$\mu = 1$ or $\mu = 5$

$(\lambda+\mu)_{max} = 13.$

4.(1) $4x - 3y + 2 = 0$

Equation of line parallel to this line

$4x - 3y + \lambda = 0$

Distance of this line from origin.

$$\frac{|\lambda|}{\cancel{5}}=\frac{3}{\cancel{5}}$$

$\lambda = \pm 3$

Equation of lines will be

$4x - 3y + 3 = 0$ ...(1)

and $4x - 3y - 3 = 0$ ...(2)

Option (1) is satisfying first line.

5.(4) $\cos^{-1} x - \cos^{-1}\left(\frac{y}{2}\right) = \alpha$

$$\cos^{-1}\left[\frac{xy}{2}-\sqrt{1-x^2}.\sqrt{1-\frac{y^2}{4}}\right]=\alpha$$

$$\frac{xy}{2}-\sqrt{1-x^2}.\sqrt{1-\frac{y^2}{4}}=\cos\alpha$$

$$\frac{xy}{2}-\cos\alpha=\sqrt{1-x^2}.\sqrt{1-\frac{y^2}{4}}$$

Squaring on both sides

$$\frac{x^2y^2}{4}+\cos^2\alpha - xy\cos\alpha = (1-x^2)\left(1-\frac{y^2}{4}\right)$$

$$\cancel{\frac{x^2y^2}{4}}+\cos^2\alpha - xy\cos\alpha = 1-x^2-\frac{y^2}{4}+\cancel{\frac{x^2y^2}{4}}$$

$$x^2+\frac{y^2}{4}-xy\cos\alpha=\sin^2\alpha$$

$$4x^2-4xy\cos\alpha+y^2=4\sin^2\alpha.$$

6.(2) $\dfrac{dy}{dx}=\dfrac{(x^2-3)-x(2x)}{(x^2-3)^2}$

$\because$ tangent is parallel to line

$$\frac{-x^2-3}{(x^2-3)^2}=-\frac{1}{3} \quad \therefore \quad \frac{x^2+3}{(x^2-3)^2}=\frac{1}{3}$$

Put $x^2=t$

$$3(t+3)=t^2+9-6t \quad \Rightarrow \quad 3t+9=t^2+9-6t \quad \Rightarrow \quad t^2-9t=0$$

$t=0,9$

$\therefore \quad x^2=0,9 \quad \Rightarrow \quad x=\pm 3$

for $\quad x=3;\ y=\frac{3}{6}=\frac{1}{2}$

for $\quad x=-3;\ y=\frac{-3}{6}=\frac{-1}{2}$

which satisfies option (2).

7.(2) $ax+y=c$ is a tangent to

$x^2+y^2=1 \qquad \ldots(1)$

and $\quad y^2=4\sqrt{2}x \qquad \ldots(2)$

equation of common tangent to (1) and (2)

$$y=mx+\frac{\sqrt{2}}{m}$$

is also tangent to (1)

So $\quad \dfrac{\left|\frac{\sqrt{2}}{m}\right|}{\sqrt{1+m^2}}=1 \quad \Rightarrow \quad \dfrac{2}{m^2}=(1+m^2) \quad \Rightarrow \quad m^4+m^2-2=0$

$(m^2+2)(m^2-1)=0$

$m=\pm 1$

Common tangent will be $y=x+\sqrt{2}$ and $y=-x-\sqrt{2}$

$y=-ax+c$ and $y=-ax+c$

On comparing $a=-1, c=\sqrt{2}$ and $a=1$ and $c=-\sqrt{2}$

$|c|=\sqrt{2}.$

8.(1) $\dfrac{dy}{dx}+y.\tan x=2x+x^2\tan x$

$\text{I.F.}=e^{\int \tan x\,dx}=e^{\log|\sec x|}=\sec x$

$y(\sec x)=\int(2x+x^2\tan x)\sec x\,dx+c \ =\int 2x\sec x\,dx+\int x^2\tan x\sec x\,dx \ =x^2\sec x+c$

$y=x^2+c.\cos x \Rightarrow y(0)=1 \Rightarrow c=1 \quad \Rightarrow \quad y=x^2+\cos x \quad \Rightarrow \quad y'=2x-\sin x$

$\Rightarrow \quad y'\left(\frac{\pi}{4}\right)=\frac{\pi}{2}-\frac{1}{\sqrt{2}} \quad \Rightarrow \quad y'\left(-\frac{\pi}{4}\right)=\frac{-\pi}{2}+\frac{1}{\sqrt{2}}$

9.(1) $|zw|=1$

$\arg\left(\frac{z}{w}\right)=\frac{\pi}{2}$

$\arg(\bar{z}w) = -\frac{\pi}{2}$

$\bar{z}w = -i$ as $|zw| = 1$

10.(3) $\frac{dv}{dt} = 50 \text{ cm}^3/\text{min}$

$v = \frac{4}{3}\pi(10+x)^3$

$\frac{dv}{dt} = \frac{4}{\cancel{3}}\pi.\cancel{3}(10+x)^2.\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{50}{4\pi(15)^2} = \frac{\overset{\overset{2}{\cancel{10}}}{\cancel{50}}}{\underset{2}{\cancel{4}}\pi.\underset{3}{\cancel{15}}.\underset{3}{\cancel{15}}} = \frac{1}{18\pi} \text{cm/min.}$

11.(3) $f = a_1 a_4 a_5$

$a_6 = 2$

$a + 5d = 2$

$= a(a+3d)(a+4d) = (2-5d)(2-5d+3d)(2-5d+4d)$

$= (2-5d)(2-2d)(2-d) = 2(2-5d)(2-d-2d+d^2)$

$= 2(2-5d)(d^2-3d+2) = 2(2d^2-6d+4-5d^3+15d^2-10d)$

$= 2(-5d^3+17d^2-16d+4) = -2[5d^3-17d^2+16d-4]$

$f' = -2[15d^2-34d+16]$

$15d^2 - 34d + 16 = 0$

$15d^2 - 24d - 10d + 16 = 0$

$3d(5d-8)(3d-2) = 0$

$d = \frac{8}{5}, \frac{2}{3}$

$f'' = -2[30d-34] = -2\left[\overset{6}{\cancel{30}} \times \frac{8}{\cancel{5}} - 34\right] = -28$

At $d = \frac{8}{5}$ given function is maximum.

12.(2) $s = \sum_{n=1}^{15} \frac{\left(\frac{n(n+1)}{2}\right)^2}{\frac{n(n+1)}{2}} - \frac{1}{2}\sum_{n=1}^{15} n$

$= \sum_{n=1}^{15} \frac{n(n+1)}{2} - \frac{1}{2}\sum_{n=1}^{15} n \quad = \frac{1}{2}\sum_{n=1}^{15} n^2 + \frac{1}{2}\cancel{\sum_{n=1}^{15} n} - \frac{1}{2}\cancel{\sum_{n=1}^{15} n} \quad = \frac{1}{2} \times \frac{15 \times 16 \times 31}{6} = 620.$

13.(3) $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -5 & 5x & 5 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & x & 1 \end{vmatrix} = 0$$

$$x(-\cancel{3x} - x^2 + \cancel{3x}) + 6(2 + x - 3) - 1(2x - 3x) = 0$$

$$-x^3 + 6x - 6 + x = 0$$

$$-x^3 + 7x - 6 = 0 \Rightarrow x^3 - 7x + 6 = 0 \Rightarrow x = 1, 2, -3$$

All three real roots $s = 0$.

14.(3) Let $I = \int (x^2)^2 e^{-x^2} x\, dx = g(x)e^{-x^2} + c$

Let $x^2 = t, 2xdx = dt$ $= \frac{1}{2}\int t^2 e^{-t} dt = \frac{1}{2}\left[-e^{-t}t^2 + \int 2t\, e^{-t} dt\right]$ $= \frac{1}{2}\left[-e^{-t}.t^2 - 2t\, e^{-t} + \int 2e^{-t}\, dt\right]$

$= \frac{1}{2}[-e^{-t}.t^2 - 2te^{-t} - 2e^{-t}]$ $= e^{-t}\left[\frac{-t^2 - 2t - 2}{2}\right] = e^{-x^2}\left[\frac{-x^4 - 2x^2 - 2}{2}\right]$

$g(x) = \frac{-x^4 - 2x^2 - 2}{2}$ $\Rightarrow$ $g(-1) = \frac{-1 - 2 - 2}{2} = -\frac{5}{2}.$

15.(3) Area of triangle $= \frac{1}{2} ac \sin \mathrm{B}$

angle $\angle \mathrm{B} = 60°$, angle are in A.P.

$\frac{\sin \mathrm{A}}{a} = \frac{\sin \mathrm{B}}{b} \Rightarrow \sin \mathrm{A} = \frac{a}{b} \times \sin \mathrm{B}$

$\frac{\sin \mathrm{C}}{c} = \frac{\sin \mathrm{A}}{a}$, $\mathrm{A} = 30°$

$a = \frac{1}{2} \times 4 = 2$, $\mathrm{C} = 90°$

area $= \frac{1}{2} \times 2 \times 4 \times \sin 60° = 2\sqrt{3}.$

16.(3) $3x^2 + 5y^2 = 32$

Tangent : $3x + 5y = 16$ $\therefore$ $Q\left(\frac{16}{3}, 0\right)$

Normal : $y - 2 = \frac{5}{3}(x - 2)$ $\Rightarrow$ $5x - 3y - 4 = 0$ $\therefore$ $R\left(\frac{4}{5}, 0\right)$

$$Ar\ \Delta PQR = \frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ \frac{16}{3} & 0 & 1 \\ \frac{4}{5} & 0 & 1 \end{vmatrix} = \frac{68}{15}.$$

17.(2) $T_{r+1} = {}^nC_r (x^2)^{n-r}\left(\frac{1}{x^3}\right)^r = {}^nC_r x^{2n-5r}$

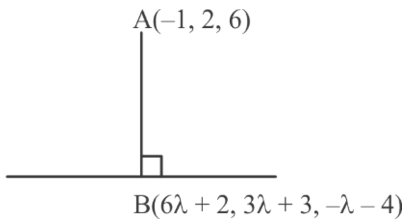
$\therefore$ $2n - 5r = 1$

$$r = \frac{2n-1}{5}$$

$$\frac{2n-1}{5} = 23 \Rightarrow n = 58$$

$$n - \left(\frac{2n-1}{5}\right) = 23 \Rightarrow n = 38 \quad \therefore \qquad n = 38.$$

18.(4) Eqn. of line : $\frac{x-2}{6} = \frac{y-3}{3} = \frac{z+4}{-4} = \lambda$



DR's of AB $\langle 6\lambda+3, 3\lambda+1, -4\lambda-10\rangle$

Given line $\perp$ AB

$\therefore \quad 6(6\lambda+3)+3(3\lambda+1)-4(-4\lambda-10)=0$

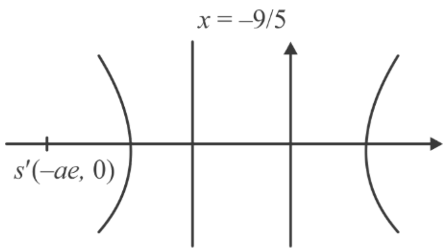
$\Rightarrow \quad \lambda = -1$

$\therefore \quad$ B(–4, 0, 0)

A(–1, 2, 6)

AB = 7.

19.(3) $\frac{x^2}{9} - \frac{y^2}{16} = 1$



$b^2 = a^2(e^2-1)$

$16 = 9(e^2-1) \Rightarrow e = \frac{5}{3} \quad \ldots(1)$

$-\frac{a}{e} = \frac{-9}{5} \Rightarrow \frac{a}{e} = \frac{9}{5}$

$\therefore \quad (-ae, 0) = (-5, 0)$

20.(2) For infinitely many solution

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3 \text{ which satisfies } \lambda^2 - \lambda - 6 = 0.$$

21.(3) $1 - P(x=0) = 1 - {}^nC_0\left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\therefore \quad \frac{1}{100} > \frac{1}{2^n}$

$2^n > 100 \quad \therefore$ Least $n = 7$.

22.(1) Negation of $\sim s \vee (\sim r \wedge s) \ = \sim s(\sim s \vee (\sim r \wedge s))$

$= s \wedge (r \vee \sim s) = s \wedge r$

23.(2) No. of beams = no. of diagonals of a 20 sided polygon

$= {}^{20}C_2 - 20 = 170.$

24.(3) $f(x) = \log_e(\sin x)$

$g(x) = \sin^{-1}(e^{-x})$

$f(g(x)) = \log(\sin \sin^{-1} e^{-x}) = -x$

$(fog)' = -1 = a$

$(fog)(\alpha) = -\alpha = b$

$a = -1$ and $b = -\alpha$ satisfies option (3).

25.(3) $\lim\limits_{x \to 1} \dfrac{x^2 - ax + b}{x - 1} = 5$

$1 - a.1 + b = 0$ ...(1)

$\lim\limits_{x \to 1} \dfrac{2x - a}{1} = 5$

$2 - a = 5 \Rightarrow a = -3$ ...(2)

From (1) and (2) $b = -4$

$\therefore \quad a + b = -7.$

26.(2) $\because \quad I = \int_{\pi/6}^{\pi/3} \dfrac{dx}{\cos^{2/3} x . \sin^{4/3} x} = \int_{\pi/6}^{\pi/3} \dfrac{\sec^2 x\, dx}{\tan^{4/3} x}$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

When $x = \dfrac{\pi}{6}, t = \dfrac{1}{\sqrt{3}}$; when $x = \dfrac{\pi}{3}, t = \sqrt{3} \quad \therefore \quad I = \int\limits_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \dfrac{dt}{t^{4/3}} = -3t^{-1/3}\Big]_{1/\sqrt{3}}^{\sqrt{3}} = 3^{\frac{7}{6}} - 3^{\frac{5}{6}}.$

27.(4) Let centre of circle $\equiv (h, k)$

$\because \quad h \geq 0, k \geq 0$

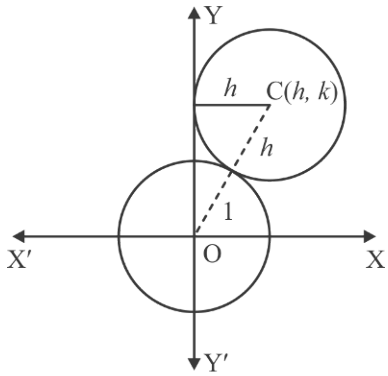
$\Rightarrow \quad h + 1 = \sqrt{h^2 + k^2}$

$\Rightarrow \quad h^2 + 2h + 1 = h^2 + k^2$

$\Rightarrow \quad k^2 = 2h + 1$

$\Rightarrow \quad k = \sqrt{2h + 1}$

$\therefore$ Locus of centre of circle $y = \sqrt{2x + 1}, x \geq 0.$



28.(2) Let $\dfrac{x-1}{2} = \dfrac{y+1}{-1} = \dfrac{z}{1} = \lambda$

Any point P on the given line is given by

$P(2\lambda + 1, -\lambda - 1, \lambda)$

$\because$ PQ is perpendicular to the plane $x + y + z = 3$.

$\therefore$ Equation of PQ : $\dfrac{x - (2\lambda + 1)}{1} = \dfrac{y + (\lambda + 1)}{1} = \dfrac{z - \lambda}{1} = k$ (say)

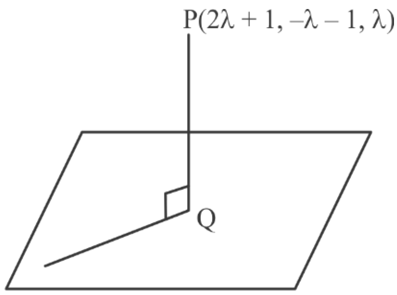
$\therefore \quad Q \equiv (k + 2\lambda + 1, k - \lambda - 1, k + \lambda).$

$\because$ Q lies on both planes $x + y + z = 3$ as well as $x - y + z = 3$.

$\Rightarrow \quad (k + 2\lambda + 1) + (k - \lambda - 1) + (k + \lambda) = 3 \Rightarrow 3k + 2\lambda = 3$ ...(1)

Also $\quad (k + 2\lambda + 1) - (k - \lambda - 1) + (k + \lambda) = 3 \Rightarrow k + 3\lambda = 1$ ...(2)

Solving (1) and (2), we get $\lambda = 0, k = 1 \quad \therefore \quad$ Point $Q \equiv (2, 0, 1)$.



29.(2) $a, b, c \rightarrow$ G.P.

Let $b = ar$ and $c = ar^2$

$\therefore \quad 3a, 7b, 19c$ is $3a, 7ar, 15ar^2 \rightarrow$ A.P.

$2.7ar = 3a + 15ar^2$

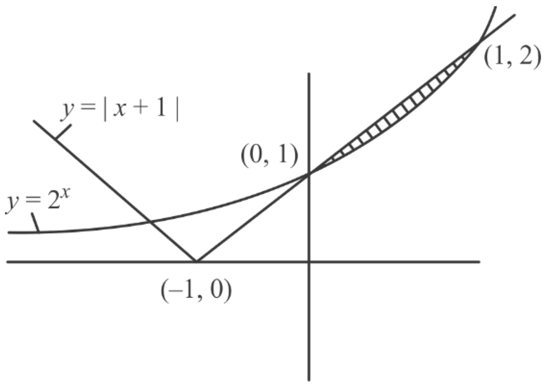
$\Rightarrow \quad r = \frac{3}{5}, \frac{1}{3}$

$\because \quad r \in \left(0, \frac{1}{2}\right] \quad \Rightarrow \quad r = \frac{1}{3}$

$d = 7a.\frac{1}{3} - 3a = \frac{-2a}{3}$

$a_4 = 3a + 3.\left(\frac{-2a}{3}\right) = a$

30.(1)



$$A = \int_0^1 (x+1) - 2^x \, dx = \frac{3}{2} - \frac{1}{\ln 2}$$